Control of Port Hamiltonian Systems by Dissipative Devices and its Application to Improve the Semi-Active Suspension Behaviour

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Abstract

The port Hamiltonian framework is a powerful tool for modeling a wide class of nonlinear systems such as robots and, more generally, mechatronic systems. The standard approaches used for the control of the port Hamiltonian systems are not applicable to a wide variety of mechatronic systems. This happens, for example, when the input control variable acts directly on some dissipative components of the system. In these cases the controlled devices can only dissipate power and the problem is to find a proper control law in order to meet the control requirements.

This paper proposes four control laws for the controlled dissipative components which allow to satisfy a set of control requirements by acting on the energy stored in a subsection of the given system or by controlling the power flowing through a physical section of the system. Although some important issues remain open, the example of the semi-active suspension shows that some positive results can be achieved by applying the proposed approach.

Key words: Nonlinear control, automotive control, semi-active suspension.

1. INTRODUCTION

From a mathematical perspective, the *Port Controlled Hamiltonian systems* (PCH) (van der Schaft, 2000) are natural candidates to model many types of real systems, as shown in the application examples cited in (Ortega *et al.*, 2002). Basically, PCH are systems defined with respect to a geometric structure capturing the basic interconnection and dissipation laws, and a *Hamiltonian* function given by the total energy stored in the system.

The control of PCH is an interesting research topic and outstanding results have already been obtained. The main results presented so far consider the possibility to operate on the power-ports of the system in order to obtain a controlled closed-loop system that is still a PCH with desired Hamiltonian function, interconnection laws and damping, see (Ortega *et al.*, 2002) and the references therein. Another approach is the control by interconnection of PCH described in (Garcia-Canseco *et al.*, 2005). However many mechatronic systems are not controlled by means of the power ports, but by operating dissipative components of the system such as variable resistors, variable dampers, clutches, electro-valves, etc. In these cases the issue is to find a proper control law that satisfies the control requirements facing the limitation that the energy can only be dissipated by the controlled components. To the best of our knowledge the problem of controlling a PCH by means of dissipative components has not already been addressed.

The key idea proposed in this paper is to divide a PCH system into two or more PCH subsystems that are connected by power preserving interconnections. The control inputs are then chosen to control the *power flowing* towards a certain subsystem or to control the *energy stored* into that subsystem. To this aim, a slight extension of the definition of PCH is proposed to allow the description of a larger set of

mechatronic systems and to obtain an explicit representation of the power flowing towards a subsystem. Thanks to the dissipative nature of the controlled components the passivity properties of the given PCH system are preserved.
The semi-active vehicle suspension (Savaresi *et al.*, 2003) is an example of mechatronic system with a controllable dissipative element. By following the proposed approach, some of the control laws already presented in literature for the semi-active suspensions are derived again and an energetic interpretation is given. Moreover, a new control law with improved performances is proposed.
The paper is organized as follows: Sec. 2 gives a brief introduction on Hamiltonian systems and extends the definition of PCH. The proposed control laws for the dissipative components are presented in Sec. 3. An application example referring to a semi-active vehicle suspension is described in Sec. 4.

2. AN EXTENSION OF THE PORT HAMILTONIAN DEFINITION

The port-Hamiltonian framework is a powerful means to model robotic, mechatronic and dynamic systems. A brief recall of some definitions written in (Van der Schaft, 2000) is given herein for reader convenience. The *Port Controlled Hamiltonian systems* have the form:

$$\begin{aligned} \dot{x} &= [J(x) - R(x)]\,\frac{\partial H}{\partial x}(x) + g(x)u \\ y &= g^T(x)\frac{\partial H}{\partial x}(x) \end{aligned} \qquad (1)$$

where

$$J(x) = -J^T(x) \quad \text{and} \quad R(x) = R^T(x) \geq 0$$

One of the key feature of PCHs is the energy perspective in modeling the physical systems. The Hamiltonian $H(x)$ represents the *energy stored* in the system and the product $y^T u$ has the physical meaning of the *power flowing* through the section characterized by power variables u and y. The power balance in (1) is the following:

$$y^T u = \frac{dH}{dt} + \frac{\partial H}{\partial x}^T R\,\frac{\partial H}{\partial x} \geq \frac{dH}{dt}$$

namely the power $y^T u$ supplied to the system is partially stored as energy in $H(x)$ and partially dissipated in element R. Many PCHs can be obtained connecting different subsystems by means of power preserving interconnections. Let (u_1, y_1) and (u_2, y_2) be the power ports of two PCHs, the general structure of a power preserving interconnection is the following:

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} 0 & A \\ -A^T & 0 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \qquad (2)$$

where matrix A can also be time-varying and/or state-dependent. With this interconnection the power flows from one system to the other without losses: $y_1^T u_1 = y_1^T A\, y_2 = y_2^T A^T y_1 = -y_2^T u_2$, namely the energy going out from one subsystem is exactly the energy going into the other.
The PCH system (1) does not consider the possibility of external inputs that directly modify the dissipation matrix $R(x)$ or the connection matrix $J(x)$. This problem was partially addressed in (Perez *et al.*, 2004) where a matrix J depending on external inputs is considered. As previously described, many mechatronic systems have dissipative components whose behaviour depends on an external input. To represent mechatronic systems as a set of PCHs connected by power preserving interconnections, the definition (1) is not enough, as shown in the example of Sec. 4. Many mechatronic systems show a direct dissipative connection between the input u and the output y. A resistor is the simplest example. The PCH system (1) cannot describe such a behaviour since the dissipation is only related to the gradient of $H(x)$. To take into account these phenomena, the following PCH has to be considered:

$$\begin{aligned} \dot{x} &= [J_1(x,v) - R_1(x,v)]\,\frac{\partial H}{\partial x}(x) + g(x,v)\,u \\ y &= g^T(x,v)\,\frac{\partial H}{\partial x}(x) - [J_2(x,v) - R_2(x,v)]\,u \\ J_i(x,v) &= -J_i^T(x,v) \qquad\qquad i = 1,2 \\ R_i(x,v) &= R_i^T(x,v) \geq 0 \qquad\quad i = 1,2 \end{aligned} \qquad (3)$$

where v is an external input vector that also may be equal to u. Matrix $J_2(x,v)$ models a power preserving interconnection (for example an ideal switch) and matrix $R_2(x,v)$ represents a dissipation element. Matrix $[J_2(x,v) - R_2(x,v)]$ has a meaning similar to matrix "D" for linear systems.

The extended definition (3) preserves all the basic properties of the PCHs and the energy perspective in modeling the physical systems. The inner product $y^T u$ has still the physical meaning of the power flowing through the port (u, y). The power balance of system (3) is the following:

$$\frac{dH}{dt} = y^T u - \frac{\partial H^T}{\partial x} R_1(x,v)\frac{\partial H}{\partial x} - u^T R_2(x,v)u \qquad (4)$$

From (4) it is straightforward to verify that (3) satisfies the energy balance equation (EBE):

$$H(x(t)) - H(x(0)) = \int_0^t y^T(\tau)u(\tau)d\tau - D(t) \qquad (5)$$

where $D(t)$ is a nonnegative function which describes all the dissipations within the system.

3. CONTROL BY DISSIPATIVE COMPONENTS

Many of the mechatronic systems described by (3) are not controlled by using the input vector u (which sometimes represents disturbances), but acting on internal dissipative components. An example is given by the semi-active suspension described in the next section, where input u is an external disturbance (the road profile velocity $\dot{x}_r$) and the controlled variable is the damping coefficient b. Further examples are the clutches (the torques on the axles are not the control inputs, only the friction torque is controlled) and some electro-valves (the main external inputs are usually the hydraulic supply pressure and the reservoir pressure). If it is not possible to modify the power flows on the power port (u, y), the approaches (Ortega *et al.*, 2002) and (Garcia-Canseco *et al.*, 2005) cannot be used. The control requirements can be often satisfied only by operating the dissipative components. This control problem, to the best of our knowledge, has never been addressed for PCH and a full result is not yet available. This paper proposes to control the dissipative components by taking into account the power exchanges between subsystems. This approach is based on two steps:

1) translate the control requirements into a desired *energy level* for a subsystem, or into a desired *input power* towards a subsystem;
2) operate the dissipative components to obtain the desired energy level or input power.

To help the solution of the first step the mechatronic system is divided into two or more subsystems of type (1) or (3), connected by a power preserving interconnection of type (2). In this way the input power and the energy stored in each subsystem can be easily computed. The correspondence between control requirements and energy levels or input powers is the target of future research and it is not addressed in this paper.

Concerning the second step, the control vector v is chosen to control the energy stored in the subsystem or the power entering the sybsystem. Four different control laws are proposed. Control laws C1 and C2 are designed to keep the entering power as close as possible to a desired level W_d. The control laws C3 and C4 are designed to control the energy H_d stored into the subsystem. To simplify the notation, let $d(v, x)$ denote the dissipated power:

$$d(v,x) = \frac{\partial H^T}{\partial x} R_1(x,v) \frac{\partial H}{\partial x} + u^T R_2(x,v)\, u \qquad (6)$$

The power balance (4) can be rewritten as follows:

$$\dot{H} = y^T u - d(v,x) \qquad (7)$$

C1) Let W_d be the desired value of the power entering a subsystem of type (1) or (3). From the power balance (7), the desired power W_d (or its closest possible value) is obtained by choosing the control vector v as follows:

$$v: \begin{cases} \mathrm{argmax}(\dot{H}+d) & \text{if } \mathrm{argmax}(\dot{H}+d) < W_d \\ \mathrm{argmin}(\dot{H}+d) & \text{if } \mathrm{argmin}(\dot{H}+d) > W_d \\ W_d = \dot{H}+d(v,x) & \text{otherwise} \end{cases} \qquad (8)$$

The last line of system (8) means that the input vector v is chosen to have $W_d = \dot{H}+d(v,x)$. In the general case the term $\dot{H}+d = y^T u$ is not easy to be computed because it requires an exact knowledge of all the system parameters. However in some applications it becomes simple, as in the example shown in Sec. 4. The first two conditions of system (8) are used when the desired power W_d cannot be obtained: in these cases the input vector v is chosen to have the input power $y^T u$ as close as possible to the desired value W_d.

C2) When the control requirement is to minimize or maximize the power W_d, control law (8) simplifies as follows:

$$\begin{cases} v: \mathrm{argmax}(\dot{H}+d(v,x)) & \text{if } \quad W_d = +\infty \\ v: \mathrm{argmin}(\dot{H}+d(v,x)) & \text{if } \quad W_d = -\infty \end{cases} \qquad (9)$$

These relations are obtained from (8) choosing $W_d = -\infty$ when power W_d is to be minimized and choosing $W_d = \infty$ when W_d is to be maximized. As shown in the next section, these control laws are much simpler to be implemented since only the maximization (or the minimization) of function $\dot{H}(x)+d(v,x)$ is required.

C3) Let H_d be the desired level of energy for the considered subsystem, let $f(z)$ be an odd function of the variable z (i.e. $f(z)\, z > 0$ if $z \neq 0$) and let function:

$$\dot{H}_s = -f(H(x) - H_d) \qquad (10)$$

denote the desired time-derivative of the energy $H(x)$ stored in the subsystem. From (7) it follows that the desired function $\dot{H}_s$ can be obtained using the following control law:

$$v: \begin{cases} \mathrm{argmax}(y^T u - d) & \text{if } \mathrm{argmax}(y^T u - d) < \dot{H}_s \\ \mathrm{argmin}(y^T u - d) & \text{if } \mathrm{argmin}(y^T u - d) > \dot{H}_s \\ \dot{H}_s = y^T u - d & \text{otherwise} \end{cases} \qquad (11)$$

The desired value $\dot{H}_s(x)$ is obtained only in the third case. The first two conditions are used when the value $\dot{H}_s(x)$ cannot be obtained: in these cases vector v is chosen to have the smallest difference.

C4) If the minimum or maximum values of the time-derivative $\dot{H}_s$ are used in (11), one obtains the following simplified control law:

$$v: \begin{cases} \text{argmax}(y^T u - d(v,x)) & \text{if } H(x) < H_d \\ \text{argmin}(y^T u - d(v,x)) & \text{if } H(x) \geq H_d \end{cases} \tag{12}$$

This control law is much simpler to be implemented, if compaerd to (11), because it requires only the maximization (or minimization) of function $y^T u - d(v,x)$.

Remark 1. As shown in equations (8) and (11), it is not ensured that the control requirements can always be satisfied by operating on the control vector v. This is mainly due to the fact that the control action of the term $d(v,x)$ may be limited in amplitude, and vector u cannot be used as control input for the system.

Remark 2. In the general case the control laws (8), (9), (11) and (12) may have many solutions for different values of the control vector v. In these cases, if a particular system structure is not given, it will not be possible to define a criterion for the choice of the best solution.

4. CONTROL OF SEMI-ACTIVE SUSPENSIONS

The semi-active suspensions are a typical example of a mechatronic system controlled by acting on a dissipative component. A quarter-car model with a semi-active suspension system is shown in Fig. 1. The damping coefficient b of the shock absorber is controlled by an electro-valve. The typical control problem is to choose the value of the damping coefficient b in order to maximize the comfort for the passengers. The ideal solution is to have the body (sprung mass) speed and acceleration as close as possible to zero in order to minimize the movements and the forces perceived by the passengers. A detailed description of the semi-active suspensions can be found in (Savaresi *et al.*, 2003) and in the references therein. This section shows how some control laws already known in literature for the semi-active suspensions can be derived again by using the proposed approach. Moreover, a new control law with slightly better performances is proposed. The variables shown in Fig. 1 have the following meanings: M_s is the quarter-car body mass, M_t is the total unsprung mass (tire, wheel, brakes, suspension links,

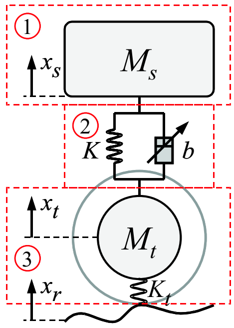

Fig. 1. Graphical representation of a quarter-car model with semi-active suspension.

etc.), b and b_d are the real and the desired damping coefficients of the shock-absorber, K and K_t are, respectively, the stiffness coefficients of the suspension spring and of the tire. Finally, x_s, x_t and x_r are the vertical positions of the body mass, the unsprung mass and the road profile, respectively. The PCH model of the system shown in Fig. 1 is the following:

$$H(x) = \frac{1}{2} M_s \dot{x}_s^2 + \frac{1}{2} M_t \dot{x}_t^2 + \frac{1}{2} K x_{st}^2 + \frac{1}{2} K_t x_{tr}^2$$

$$\begin{bmatrix} \ddot{x}_s \\ \ddot{x}_t \\ \dot{x}_{st} \\ \dot{x}_{tr} \end{bmatrix} = \begin{bmatrix} \frac{-b}{M_s^2} & \frac{b}{M_s M_t} & \frac{-1}{M_s} & 0 \\ \frac{b}{M_s M_t} & \frac{-b}{M_t^2} & \frac{1}{M_t} & \frac{-1}{M_t} \\ \frac{1}{M_s} & \frac{-1}{M_t} & 0 & 0 \\ 0 & \frac{1}{M_t} & 0 & 0 \end{bmatrix} \begin{bmatrix} M_s \dot{x}_s \\ M_t \dot{x}_t \\ K x_{st} \\ K_t x_{tr} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ -1 \end{bmatrix} \dot{x}_r$$

$$y = \begin{bmatrix} 0 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} M_s \dot{x}_s & M_t \dot{x}_t & K x_{st} & K_t x_{tr} \end{bmatrix}^T$$

The gravitational forces have been compensated by the springs pre-load and they do not appear in the equations. The state variables $x_{st} = x_s - x_t$ and $x_{tr} = x_t - x_r$ represent, respectively, the deformations of the spring and of the tire with respect to the equilibrium length.

The dissipation coefficient b depends on the actuator dynamics. The actuator is usually described by a first order linear system with saturation:

$$\dot{b} = \begin{cases} 0 & \text{if} \quad b = b_{max} \text{ and } b_d \geq b_{max} \\ 0 & \text{if} \quad b = b_{min} \text{ and } b_d \leq b_{min} \\ \beta\,(b_d - b) & \text{else} \end{cases}$$

where b_d is the desired damping, $b_{max} > b_{min} > 0$ and $\beta > 0$ is the bandwidth of the actuator.

Remark 3. For the sake of clarity the described suspension system is linear as in (Savaresi *et al.*, 2003). For a real suspension system both K and b are nonlinear functions of the state variables. However the results presented in the following section hold also for the nonlinear case.

4.1. *Partition of the PCH*

The semi-active suspension system can be partitioned in the following three connected PCHs (the dashed boxes of Fig. 1):

1) Subsystem 1, sprung mass PCH:

$$\begin{aligned} H_1 &= \frac{1}{2} M_s \dot{x}_s^2 \\ \ddot{x}_s &= \begin{bmatrix} 0 \end{bmatrix} M_s \dot{x}_s + \begin{bmatrix} \frac{1}{M_s} \end{bmatrix} u_1 \\ y_1 &= \begin{bmatrix} \frac{1}{M_s} \end{bmatrix} \frac{\partial H_1}{\partial \dot{x}_s} = \dot{x}_s \end{aligned} \tag{13}$$

2) Subsystem 2, spring-damper PCH:

$$\begin{aligned} H_2 &= \frac{1}{2} K x_{st}^2 \\ \dot{x}_{st} &= \begin{bmatrix} 0 \end{bmatrix} K x_{st} + \begin{bmatrix} 1 & -1 \end{bmatrix} \begin{bmatrix} u_{2,1} \\ u_{2,2} \end{bmatrix} \\ \begin{bmatrix} y_{2,1} \\ y_{2,2} \end{bmatrix} &= \begin{bmatrix} 1 \\ -1 \end{bmatrix} K x_{st} + \begin{bmatrix} b & -b \\ -b & b \end{bmatrix} \begin{bmatrix} u_{2,1} \\ u_{2,2} \end{bmatrix} \end{aligned} \tag{14}$$

3) Subsystem 3, wheel and tire PCH:

$$\begin{aligned} H_3 &= \frac{1}{2} M_t \dot{x}_t^2 + \frac{1}{2} K_t x_{tr}^2 \\ \begin{bmatrix} \ddot{x}_t \\ \dot{x}_{tr} \end{bmatrix} &= \begin{bmatrix} 0 & \frac{-1}{M_t} \\ \frac{1}{M_t} & 0 \end{bmatrix} \begin{bmatrix} M_t \dot{x}_t \\ K_t x_{tr} \end{bmatrix} + \begin{bmatrix} \frac{1}{M_t} & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} u_{3,1} \\ \dot{x}_r \end{bmatrix} \\ \begin{bmatrix} y_{3,1} \\ y_{3,2} \end{bmatrix} &= \begin{bmatrix} \frac{1}{M_t} & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} M_t \dot{x}_t \\ K_t x_{tr} \end{bmatrix} = \begin{bmatrix} \dot{x}_t \\ -K_t x_{tr} \end{bmatrix} \end{aligned} \tag{15}$$

The three subsystems are connected by the following power preserving structures:

$$\begin{bmatrix} u_{2,1} \\ u_1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} y_{2,1} \\ y_1 \end{bmatrix} \rightarrow \begin{bmatrix} u_{2,1} \\ u_1 \end{bmatrix} = \begin{bmatrix} \dot{x}_s \\ -K x_{st} - b \dot{x}_{st} \end{bmatrix}$$

$$\begin{bmatrix} u_{2,2} \\ u_{3,1} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} y_{2,2} \\ y_{3,1} \end{bmatrix} \rightarrow \begin{bmatrix} u_{2,2} \\ u_{3,1} \end{bmatrix} = \begin{bmatrix} \dot{x}_t \\ K x_{st} + b \dot{x}_{st} \end{bmatrix}$$

4.2. *Passive suspensions*

Although the passive suspensions are not controllable, their behavior from a power/energy perspective is now analyzed to get some insight about the proposed approach. The energy stored in the spring-damper subsystem 2 is:

$$H_2 = \frac{1}{2} K x_{st}^2 \geq 0$$

and the power $y_2^T u_2$ entering the subsystem is:

$$y_2^T u_2 = \dot{H}_2 + d_2(b, x) = K x_{st} \dot{x}_{st} + b \dot{x}_{st}^2 \tag{16}$$

If the control requirement is to dissipate in the subsystem as much power as possible, then the control law (9) with $W_{2d} = +\infty$ can be used:

$$b : \text{argmax}(K x_{st} \dot{x}_{st} + b \dot{x}_{st}^2) \quad \rightarrow \quad b_d = b_{max}$$

that is the damping coefficient b should be constant at its maximum value, $b_d = b_{max}$, and therefore a passive suspension is enough to meet the requirement. However it is well known that this solution is not optimal both for handling and for comfort.

4.3. *Classic two-state sky-hook control*

The target of the sky-hook control is to keep the body speed $\dot{x}_s$ and the body acceleration $\ddot{x}_s$ as close as possible to zero facing the road profile $\dot{x}_r$. The classic two-state "sky-hook" control law known in literature, see (Savaresi *et al.*, 2003), is:

$$b_d = \begin{cases} b_{max} & \text{if} \quad \dot{x}_{st} \dot{x}_s \geq 0 \\ b_{min} & \text{else} \end{cases} \tag{17}$$

The kinetic energy $H_1(\dot{x}_s)$ of subsystem 1, see (13), is always positive. Let us consider the control law (12) with zero energy level for the body: $H_{1d} = 0$. In this case we have:

$$y_1^T u_1 - d_1(b, x) = -b \dot{x}_s \dot{x}_{st} - K x_{st} \dot{x}_s$$

and only the second condition of (12) is possible. Function $y_1^T u_1 - d_1(b, x)$ can be minimized only by minimizing the term $-b \dot{x}_s \dot{x}_{st}$. This leads to the same condition $b_d = b_{min}$ given by control law (17).

4.4. *Acceleration-Driven-Damper control*

The Acceleration-Driven-Damper (ADD) control is proposed in (Savaresi *et al.*, 2003). Under mild assumptions, it has been demonstrated that ADD control is "optimal" in the sense that it minimizes the vertical body acceleration $\ddot{x}_s$ when no road-preview is available. The ADD control is defined as follows:

$$b_d = \begin{cases} b_{max} & \text{if} \quad \ddot{x}_s \dot{x}_{st} \geq 0 \\ b_{min} & \text{else} \end{cases} \tag{18}$$

This control law can also be obtained, in an alternative way, by using the control law C2 given in (9).

Consider the subsystem 2 described in (14). Let the desired power W_{2d} be the following:

$$W_{2d} = \begin{cases} +\infty & \text{if} \quad y_2^T\, u_2 \leq 0 \\ -\infty & \text{if} \quad y_2^T\, u_2 > 0 \end{cases} \tag{19}$$

where $y_2^T\, u_2 = K\, x_{st}\, \dot{x}_{st} + b\, \dot{x}_{st}^2$ is the power entering the subsystem, see eq. (16). Consequently the control requirement is to keep the entering power $y_2^T\, u_2$ as close as possible to zero: if $y_2^T\, u_2 > 0$ ($y_2^T\, u_2 < 0$) the requested power W_{2d} is the lowest (highest) possible. This control law mimics a sort of sliding mode control of the power. According to (9) and (19) the desired damping coefficient b_d must be chosen as follows:

$$b_d = \begin{cases} b_{max} & \text{if} \quad y_2^T\, u_2 \leq 0 \\ b_{min} & \text{else} \end{cases} \tag{20}$$

This control is exactly the same as in (18) since:

$$y_2^T\, u_2 = (K\, x_{st} + b\, \dot{x}_{st})\, \dot{x}_{st} = -M_s\, \ddot{x}_s\, \dot{x}_{st} \tag{21}$$

The last two examples show that the control laws C2 and C4 may be simple to be implemented since only a partial knowledge of the system state is required.

4.5. *Power-Driven-Damper control*

Control law (18) may show an oscillating behavior on b_d if the bandwidth β is wide enough and if $b_{max}\, \dot{x}_{st}^2 + K\, x_{st}\, \dot{x}_{st} > 0$ and $b_{min}\, \dot{x}_{st}^2 + K\, x_{st}\, \dot{x}_{st} < 0$. This is due to the direct dependence of $\ddot{x}_s$ (or y_2) on the damping coefficient b, namely if $\beta \to \infty$ the controlled variable b affects instantaneously the measured variable $\ddot{x}_s$.
From (19) (but not from (18)!) it is clear that the ADD control mimics a sort of sliding mode control of the power whose aim is to steer the power $y_2^T\, u_2$ to zero. This observation leads to an alternative control strategy which can be obtained from control law C1, see (8), when the requirement $W_{2d} = 0$ is considered. Matching (8), (16) and (21) we obtain the following new Power-Driven-Damper (PDD) control law:

$$b_d = \begin{cases} b_{max} & \text{if } K x_{st} \dot{x}_{st} + b_{max} \dot{x}_{st}^2 < 0 \\ b_{min} & \text{if } K x_{st} \dot{x}_{st} + b_{min} \dot{x}_{st}^2 \geq 0 \\ (b_{max} + b_{min})/2 & \text{if } \dot{x}_{st} = 0 \quad \text{and} \quad x_{st} \neq 0 \\ -K x_{st}/\dot{x}_{st} & \text{otherwise} \end{cases} \tag{22}$$

The first two equations in (22) lead to the same behaviour as in (19). The last two equations deal with the problem of the oscillations: the desired damping b_d is indeed set to obtain exactly $W_2 = 0$, namely it equals the equivalent control b_s in the sliding mode sense. This value b_s belongs to the interval $[b_{min}\ ,\ b_{max}]$. When $\dot{x}_{st} = 0$ the power $W_2 = y_2^T\, u_2$ equals the desired value $W_{2d} = 0$ and the control requirement is satisfied for any damping b. In this case the desired damping b_d is set to the average damping value during the transients ($x_{st} \neq 0$) and it is set to the minimum in the steady state condition ($x_{st} = \dot{x}_{st} = 0$).
The advantages of the proposed PDD control are clear from the simulation results shown in the next section. The cost is the need for the knowledge of the spring stiffness K and for the exact control of the damping coefficient b, both required by (22).

4.6. *Simulation results*

The behaviour of the PDD control is compared to the ADD control in Figs. 2, 3 and 4. Since the ADD control is almost optimal in terms of body acceleration minimization (maximum comfort) the comparison does not take into account the other less efficient control strategies. This comparison can be found in (Savaresi *et al.*, 2003) and in (Savaresi *et al.*, 2004). The parameters for the simulations and the comfort evaluation method are the same as in (Savaresi *et al.*, 2004).
The comparison of the approximated frequency responses is shown in Fig. 2. The lower is the frequency response the better is the control algorithm: the ADD is slightly better in the frequency range from 2 to 10 Hz, conversely the PDD is slightly better at low and high frequencies.
The time responses shown in Figs. 3 and 4 underlines the advantages of the PDD control in terms of lower jerk for both sinusoidal and step road profiles. Concerning the jerk, the improvement is evident. This improvement is not paid in terms of a worsening of the body acceleration, conversely for both the time responses the acceleration behaviors are quite similar: the PDD is slightly better in the step response while the ADD is slightly better for the tone response.

5. CONCLUSIONS

The paper addressed the problem of controlling a port Hamiltonian system by operating on its dissipative coefficients. The key idea is to divide the Hamiltonian system into two or more subsystems that are connected by power preserving interconnections.
The control inputs have been chosen to control the stored energy or the dissipated power of certain particular subsystems. Only some preliminary results have been presented, many other problems and ques-

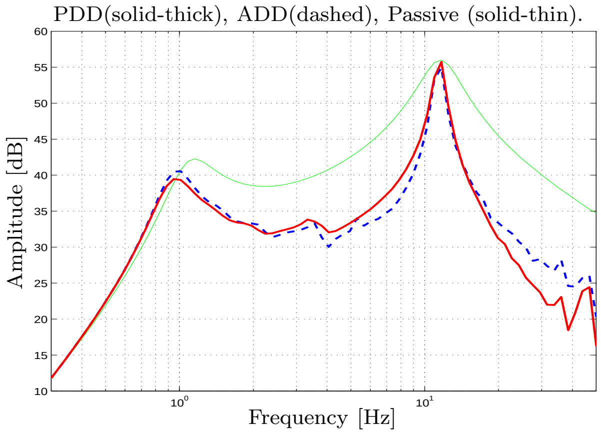

Fig. 2. Approximated frequency response from disturbance x_r to acceleration $\ddot{x}_s$: PDD control, ADD control and Passive suspensions.

tions remain open. Nevertheless, the semi-active suspension example has shown that some positive results can be achieved by applying the proposed approach.

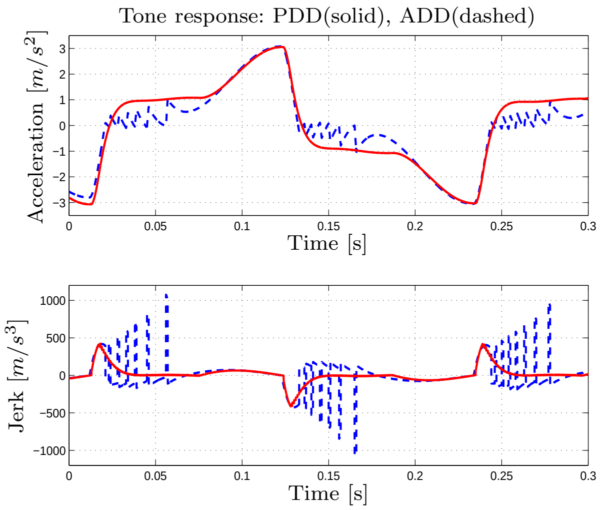

Fig. 3. Time response to a pure 4.5 Hz sinusoidal road profile: body acceleration (top) and jerk (bottom) for PDD (solid) and ADD (dashed).

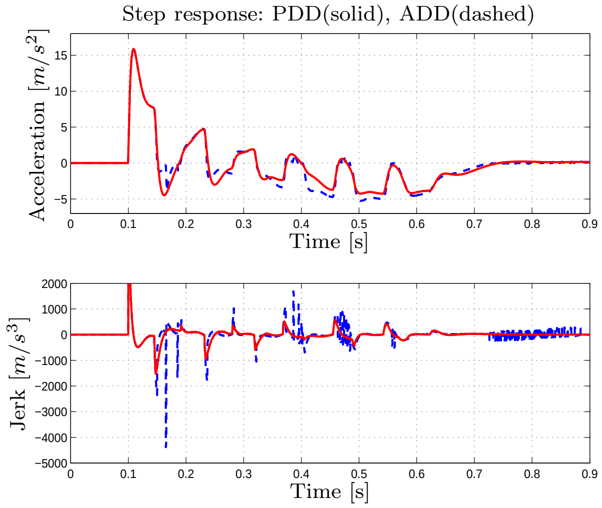

Fig. 4. Time response to a step road profile: body acceleration (top) and jerk (bottom) for PDD (solid) and ADD (dashed) controls.

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